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PHYSICS

Gravity Mediated Interaction Cross Sections Without Ultraviolet Divergence.

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ABSTRACT

In this paper I will show how to calculate gravitationally corrected QFT interaction cross sections without ultraviolet divergences. I will also make a prediction for the result of a simple experiment proposed in this paper. This work builds on papers published in 2014 and a talk given at the April 2015 APS conference in which I described relativization. In relativization QFT's are made to comply with the precepts of General Relativity. This is different than the quantization that is cannon in the physics community, however I have numerical results already published which show it has promise. Here I show how this model has predictive power for experiments in this universe, and no other.

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REVIEW

In a series of three papers published in the summer of 2014 and early winter of 2015 the basics of Relativization were laid out. Here is a brief review, for a full treatment please see (Farmer 2014a) (Farmer 2014b) (Farmer 2015a) (Farmer 2015b).

The fundamentals of this model are encapsulated in the following axioms. These axioms are not all my own invention. These are based on the knowledge built up by the work of many, cited where appropriate, for how nature should behave in light of relativity and Quantum Field Theory. Other axiomatic formulations for QFT in curved space time have been published such as (Hollands and Wald 2015) they dealt with a static curved background. What follows will give a formulation which can admit a dynamical background.

1. The principle of Relativization: All physical theories must obey the Einstein Equivalence Principle. “that for an infinitely small four-dimensional region, the relativity theory is valid in the special sense when the axes are suitably chosen.” (Einstein 1916) In other words, physical theories must be formulated in a way that is locally Lorentz covariant and globally diffeomorphism covariant.

2. Spectrum condition: All possible states of a QFT will be in the Fock-Hilbert space H . An operator on H must map states to other states in H.

3. Normalization condition: The inner product on Hilbert space must be in a set isomorphic to the division algebras R,C,H, O. (Baez 2012). For example, an inner product on H of the form $<\psi|\psi> = j^a$ with $j^a \in$ Minkowski space-time and $|\psi> \in$ H.

4. The principle of QFT locality: QFT interactions occur in the locally flat space at the point of interaction. The propagation of particles between interactions is governed by relativity.

5. Specification condition: Relativized QFT's are defined by the above and the tensor product of their state space with Minkowski space, and the algebra A of operators on H. For a theory T, $T = \{H, H \otimes M, A(H)\}$ (Inspired by a similar statement in (Hollands and Wald 2015).

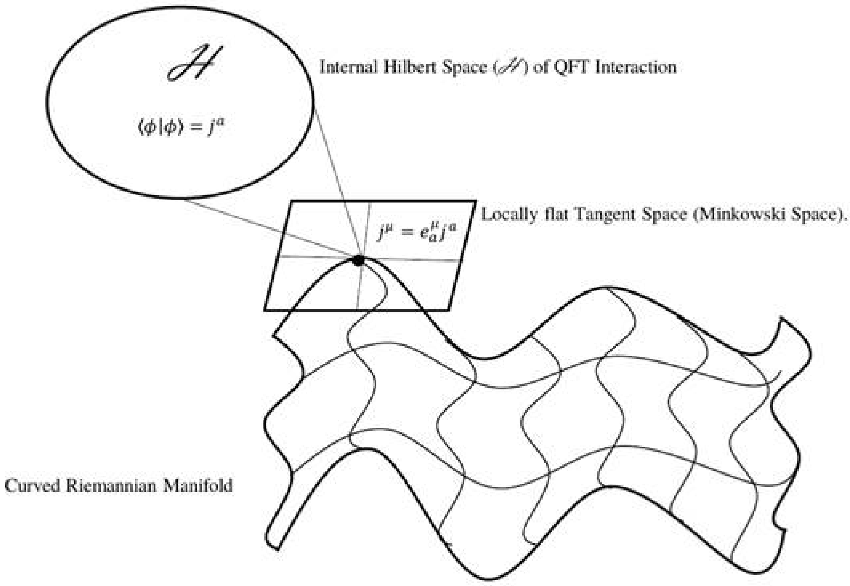

Figure 1: A visualization of the relationship between the Hilbert space of a quantum system at a given point in space-time, the local flat tangent space to the underlying Riemannian manifold, and the Riemannian manifold. It can be said that in relativization theory the vector quantities we observe in nature are projections of Hilbert space onto the tangent space-time near a point. Curvature of space time reacts to and relates these vectors at one point to vectors at another point. The Hilbert space tells the Riemannian manifold how to curve, and the curved manifold tells Hilbert space what the curvature is.

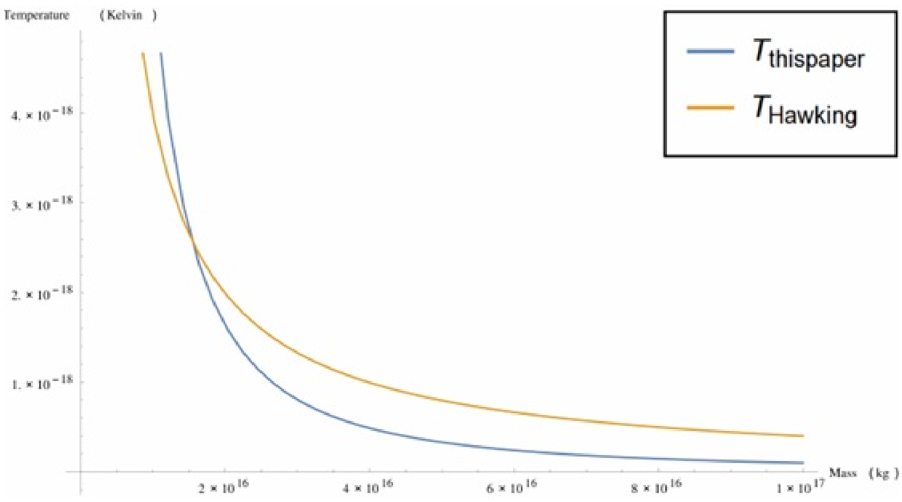

Figure 2: Black Hole Temperature as a Function of Mass. Here is a plot of the famous result due to Stephen Hawking along with a result from my own model (Farmer, Quantum gravity by relativization of Quantum Field Theory. 2014a) (Farmer, Fundamentals of Relativization 2014b). In this model black holes produce Hawking radiation at a rate which is compatible with observations yet differs from Hawking's prediction. To see which is correct would take so long as to be impractical. So I will propose a simple experiment.)

I applied these axioms to the problem of black hole thermodynamics and arrived at a numerical model for the temperature-mass relationship of a black hole which agrees with Hawking. In this model black holes neither blow up nor last forever in agreement with what we all know about black holes at present. I will now apply these axioms to the problem of computing interaction cross sections.

FINITE GRAVITY MEDIATED INTERACTION CROSS SECTIONS.

Following the axioms, I will compute the interaction cross section of graviton-graviton interaction. The fifth axiom can be satisfied by first writing down a Lagrangian for the system.

Where R is the scalar curvature operator, *Rab* is the curvature tensor, and *φ* is the Higgs field. These are all operators in the Hilbert space. Their derivation is given in (Farmer 2014b).

$$R_{ab} = (d\bar{\phi}\gamma_a\phi \wedge \gamma_b + \bar{\phi}\gamma_a\phi \wedge \gamma_c \wedge \bar{\phi}\gamma^c\phi \wedge \gamma_b)\phi\bar{\phi}$$ Equation 1

I will use a Lagrangian published previously and it shall be referred to as the relativized extended standard model (RESM). Presented here in a brief form as equation 2.

$$L_{RESM} = \sqrt{-g}\left(R - \frac{1}{6}\bar{\phi}\gamma^a R_{ab}\phi\gamma^b + L_{SM}\right).$$ Equation 2

Before I proceed I will rewrite this Lagrangian in terms of exterior algebra and treat the operators in it as differential forms. I will also use the geometric product as defined in (Somaroo, Lasenby and Doran 1999). This is easy to confuse for an inner product as in the standard notation both lack a symbol. The result is as follows

$L_{RESM} = T^{ab}R_{ab} + T^{ab} \wedge R_{ab} + L_{SM} = T^{ab}R_{ab} + L_{SM}$, where $T^{ab} = R^{ab} \wedge 1 + \frac{1}{6}\gamma^a\bar{\phi} \wedge \gamma^b\phi \wedge 1$, and the geometric product has been used.

The next step is to compute the generating functional for this model $Z_0 = e^{\gamma^1 s}$. (In this framework it makes sense to user the gamma matrices as a quaternion basis. Using the gamma matrices in this way has several geometric algebraic advantages.) For now I will drop L_{SM} because its generating functionals are well known. To compute the generating functional I need to simplify this integral $\gamma^1 s = \gamma^1 \int d^4x T^{ab} R_{ab}$. After integration by parts and full simplification the integral works out to $\gamma^1 s = \gamma^1(\eta^{ab}(2 + \frac{2}{3}\delta(x - x')))R_{ab}$. Which can be rewritten as $\gamma^1 s = \widetilde{T^{ab}} R_{ab}$. Therefore, the generating functional will be given by.

$$Z_0 = \frac{e^{\gamma^1 \widetilde{T^{ab}} R_{ab}}}{\gamma^1 \widetilde{T^{ab}}} = \gamma^1 \widetilde{T_{ab}}^{-1} e^{\gamma_1 \widetilde{T^{ab}} R_{ab}}$$ Equation 3

Consider equation 3. If the curvature goes to infinity, then the generating functional oscillates rapidly. If the separation between points x and x' is zero, i.e. a system of zero length the generating functional remains finite. Put another way, even with a wavelength of zero the amplitude of the oscillation remains finite. Following a similar procedure of integration by parts one can find $Z1$.

$$Z_1 = \frac{e^{\gamma^1 \widetilde{T^{ab}} \wedge R_{ab}}}{\gamma^1 \widetilde{T^{ab}}} = \gamma^1 \widetilde{T_{ab}}^{-1} e^{\gamma_1 \widetilde{T^{ab}} \wedge R_{ab}}$$

Equation 4

As per the axioms of relativization this quantity is a quaternion, which is one of the groups that a quantum mechanics must have as its set of "scalars". To get the observed cross section for graviton-graviton interaction in this model I need to take some functional derivatives with respect to T_{ab}. This works out to a simple expression in terms of the scalar of the stress energy tensor

$$Z_0^{-1}\left(-\gamma_1 \frac{\delta}{\delta \widetilde{T^{ab}}}\right)\left(\gamma_1 \frac{\delta}{\delta \widetilde{T^{ab}}}\right) Z \rightarrow 2\frac{1}{T^2} + \gamma^1 (T^2 - 1).$$

Equation 5

By examination one can see that this equation will not give an infinite answer for a finite input. One could set T equal to the scalar stress energy of the whole universe and get a large, but finite answer. One could set it equal to the cosmological vacuum energy and get a finite answer. Using this framework one may compute the gravitationally corrected interaction cross sections for all standard model particles by adding the stress energy due to the standard model $\widetilde{T_{ab}} = \widetilde{T_{ab}} + \widetilde{T_{ab}}_{SM}$ and computing the following.

$$\sigma = \left(Z_{0SM}^{-1}\left(-\gamma_1 \frac{\delta}{\delta J_{SM}}\right)\left(\gamma_1 \frac{\delta}{\delta J_{SM}}\right) Z_{SM} Z_1\right)^2$$

Equation 6

PROPOSED EXPERIMENT.

The three-inch equation, equation 6, suggest an experiment. It can be rewritten in the following approximate form.

$$\sigma \approx \sigma_{SM}(Z_1)^2 \sim \sigma_{SM}\left(1 + l_p^2 r^{-2} + l_p^4 r^{-4} + \cdots\right)^2$$

Equation 7

The experimental prediction of equation 7 is for a very small difference in the interaction cross sections of particle interactions which will vary with radial distance from a large gravitating mass. The effect predicted would be in addition to time dilatation and gravitational time dilation. This effect would not turn up unless identical particle physics experiments were performed at a small separation in r, it would have to be on the order of a meter or less. The smaller the separation, the greater the effect. The effect would be of order one at $r = l_p$. NOTE: This does not have to be close to the gravitational source. The experiment has to be moved incrementally closer or father from the source. This experiment would work at any distance. The effect would be stronger closer to a strong gravity source. Earth itself will suffice.

The simplest standard model interaction to use for this experiment would be radioactive decay. A sample could be placed at, say, 50 cm and allowed to decay, then lowered in regular intervals the smaller the intervals the greater the effect. The smaller the intervals the greater the number of data points. *I will endeavor to perform just such an experiment and report the results in a future paper.*

DISCUSSION

This paper builds on a series of papers published in 2014 and discussed at the April 2015 APS meeting (Farmer 2015b) . I have shown that in the framework of relativized quantum field theory, gravitational effects in interactions may be calculated with no ultraviolet divergence at any length scale. For this extraordinary claim I rely on previously published numerical work, the functional integrals in this paper, and experimental predictions made in this paper. Mathematics can model many things while being internally consistent. Only an experiment can really “prove” what is what. So I will not speak of proof but only of numerical models and mathematical hypotheses which demand testing, and an experiment to effect that testing.

For this model I also borrow an argument made lately in support of super string theory, and M-Theory (Becker 2015). That is, when a model incorporates into itself previously existing well tested models, that new model gains those observations as experimental support. Like string/M-theory the model described in the previous papers (Farmer 2014a) (Farmer 2014b) and in this paper, incorporates General Relativity and the Standard Model of particle physics. Therefore, it also gains the deep catalog of experimental, and even astronomical observations in support of those theories. If that reasoning works for M then it works for the model in this paper.

What this model predicts that is new and verifiable or falsifiable are the cross sections for graviton-graviton scattering by way of graviton exchange without ultraviolet divergence. I have demonstrated this mathematically by application of the generating functional technique.

CONCLUSION

The question that remains is are these equations accurate to nature, do they predict what the gravitational cross sections actually are? Only time and experimentation will settle that. I have proposed and will attempt to perform just such an experiment.

I know this much; nonsense would not model nature at all. In one paper last year I showed that this model predicts Hawking radiation. The one solid touchstone of so-called “quantum gravity”. In this paper I have proposed a very simple experiment that can test an aspect of this model of relativized quantum field theory.

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APPENDIX

A very well informed comment by Douglass Sweetser points out that graviton-graviton scattering is not a very common occurrence based on how hard it has been to observe gravitational waves. This is a very valid point. Very sensitive experiments such as have searched for them directly and indirectly. The observation that shows most promise so far is the POLARBEAR collaborations observation of the CMB which did observe B-Mode Polarization (The Polarbear Collaboration 2014). What's more, they are a prediction of Einstein's General Relativity. Theorist, in general, don't bet against Einstein's relativity. Most models of quantum gravity assume something like a graviton. This is not "quantum gravity" in the sense of quantized General Relativity but the interaction of one gravitational field with another and itself exist in this model. I choose to call that a graviton interaction.