BACKGROUND

I received my Bachelor of Science in Economics from the WP Carey school of business at Arizona State University. During my time at ASU I took Dr Ted Solis’ Gamelan Ensemble class twice, the first time for a necessary humanities credit and the second time for my own pleasure. I am currently not a student at ASU, but Dr Solis has been generous enough to let me participate in his fall 2015 Gamelan Ensemble class.

I was not a student of music, but I am an everyday musician and I have self studied various areas of music. I would estimate that I have read about 1,500 pages of academic music material. For this paper, Harry Partch’s ‘Genesis of a Music’ has been most insightful, providing me with a good understanding of the history, mechanics, and mathematics of tuning systems from around the world.

BRIEF OVERVIEW OF GAMELAN MUSIC

Gamelan is a traditional style of music from Indonesia. The word ‘Gamelan’ translated into English roughly means ‘Orchestra’, but within this paper ‘Gamelan’ will refer to the music culture itself, rather than the collection of instruments. To refer to the collection of instruments I will use ‘Gamelan orchestra’. Gamelan orchestras are percussive, and largely contain gongs and metallophones[1].

Gamelan orchestras, quite unlike the ensembles in other music cultures, are each uniquely tuned. There is no Gamelan-wide tuning standard, only orchestra specific tunings. This means that one could not use a Gamelan instrument from one orchestra in a another orchestra, for the instruments would not be in tune with each other. This is in contrast with European music culture, where every instrument, regardless as to what ensemble it plays in, is tuned the same. One could say, European tuning is a culture wide standard, and Gamelan tuning is an ensemble wide standard.

In addition to the great variation of tunings between Gamelan orchestras, the tuning is exclusively done by a religious class of people in Indonesia who keep essential aspects of Gamelan orchestra production a trade secret. There is no body of work from the instrument makers explaining the theory
behind Gamelan tuning.

While the tuning of Gamelan orchestras are various and mysterious, there are norms and consistencies that span Gamelan music. For example, all Gamelan instruments are in one of two tuning systems, called Slendro and Pelog. Slendro and Pelog are distinct and separate collections of tones, unlike European scales, which are subsets of a larger collection of tones (C major being an 8 tone subset of the greater 12 tones).

INSTRUMENTATION

Within this paper we will examine the exact frequencies of two instruments called Sarons. The Sarons examined are both in the Slendro tuning system. Each Saron spans one octave, and are one octave apart. The higher and lower octave Sarons are called Saron Barung, and Saron Demung, respectively. Each Saron contains 6 bronze bars, notated as 1, 2, 3, 5, 6 and 1.[ii][iii] Sarons act as the primary melodic instrument in Gamelan orchestras, and come in varieties that are in higher octaves than the Barung, and lower than the Demung.

METHOD FOR FREQUENCY DETERMINATION

Recordings were collected from the Javanese Gamelan instruments at Arizona State University with the help and permission of Dr. Ted Solis. Starting ten minutes before a Gamelan Ensemble class, Dr Solis and I recorded two short samples, one from the Saron Barung, and one from the Saron Demung. During each sample Dr Solis would twice play through each of the 6 bars of Slendro scale, ascending from the bottom, and letting each tone ring for about 2 seconds. After each tone, Dr Solis would silence the prior bars to ensure that each bar sounded unaccompanied by the other bars.

I used the audio software Audacity to determine the frequencies of each tone in the Slendro scale. Rather than rely on Audacitys frequency determination processes, which I have found to be unreliable, I determined each frequency by a guess and check process using my ear. For each tone in the Slendro scale, I would generate a sine wave of comparable volume, and arbitrary frequency. I repeatedly played the slendro tone, and the sine wave concurrently, while adjusting the frequency of the sine wave until it was perceptually identical to that of the slendro tone. The process was is analogous to how two violinists might tune the strings of one violinists violin, to the strings on the other violinists violin, however instead of strings, it was between a recording of a Saron, and a sine wave of a known frequency.

MARGIN OF ERROR IN DETERMINING SLENDRO FREQUENCIES

To detect if the Gamelan recording, and the sine wave were in tune, I would observe whether or not there was ‘beating’ between the two sounds. Beating is the auditory phenomenon of two tones very close in pitch, oscillating between being in and out of phase with each other.[iv].

two sine waves with very close frequencies beating between doubled volume and silence

two sine waves with very close frequencies beating between doubled volume and silence

The closer two tones are to each other in frequency, the longer it takes for a beat to occur between them. As the difference between two frequencies approaches zero, the duration of a beat approaches infinity. Because beats happen on such slow time scales, they are individually perceivable and countable.

If a human being detects frequency differences by the presence of beating, then, roughly speaking, a beat of a duration greater than twice the duration of the tone sounding cannot be detected by a human being. Each bar in the samples was sounded for about 2 seconds, which puts an upper limit on a human listeners capacity to detect frequency differences. The duration of beating for two nearby frequencies can be determined by the following equation
The beat duration between 700 hertz and 699.7 hertz, for example is therefore 3.33 seconds. The theoretical upper limit for human frequency detection for 2 second soundings is 0.25 hertz.\[v\]

The theoretical upper limit for a human listeners capacity to discern frequency difference via beat detection, coheres well with the experience I had while guessing and checking the Sarons tones. I was not able to hear beats after reaching one decimal place of accuracy for each of the frequencies.

MEASUREMENTS

Below are the recorded frequencies of the Barung and the Demung

<table>
<thead>
<tr>
<th>Note</th>
<th>Barung</th>
<th>Demung</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>524.5</td>
<td>261.5</td>
</tr>
<tr>
<td>2</td>
<td>600.3</td>
<td>301.5</td>
</tr>
<tr>
<td>3</td>
<td>688</td>
<td>343.4</td>
</tr>
<tr>
<td>5</td>
<td>803</td>
<td>399</td>
</tr>
<tr>
<td>6</td>
<td>918.6</td>
<td>456.5</td>
</tr>
<tr>
<td>1</td>
<td>1055.5</td>
<td>524.5</td>
</tr>
</tbody>
</table>

ANALYSIS

OCTAVE

For both the Barung and the Demung, the octave tone is a few hertz higher than the theoretical octave, which would be the frequency times 2. This is called octave stretching, and is a normal feature of tuned instruments. By octave stretching, a scales tones correspond closer to how harmonics (such as the octave) actually sound, rather than their theoretical values\[vi\].

TUNING SYSTEM

To assess the Slendro scale, and make an attempt at determining the theoretical basis behind the measured frequencies of Slendro instruments, we first must express the measured frequencies not as hertz, but as intervals between musical notes. In this paper we assess intervals in absolute terms\[vii\], meaning every tone will be compared with the base frequency of the scale. In our absolute comparison, we will treat the 1 note in the Slendro scale as our base frequency\[viii\]. Calculating absolute intervals is done by dividing every notes frequency by the frequency of the base note.

Listed below are the absolute intervals for the Barung and Demung.

<table>
<thead>
<tr>
<th>Note</th>
<th>Barung</th>
<th>Demung</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1.144518589</td>
<td>1.152198853</td>
</tr>
<tr>
<td>3</td>
<td>1.311725453</td>
<td>1.313193117</td>
</tr>
<tr>
<td>5</td>
<td>1.530981888</td>
<td>1.52581262</td>
</tr>
<tr>
<td>6</td>
<td>1.751382269</td>
<td>1.745697897</td>
</tr>
<tr>
<td>1</td>
<td>2.012392755</td>
<td>2.005736138</td>
</tr>
</tbody>
</table>
EQUAL TEMPERAMENT

It is widely understood that the Slendro scale is comprised of 5 tones, that are more or less equally spaced through the octave. We call tuning systems, in which the tones are logarithmically equally distributed through the octave equal tempered scales (logarithmically, because people perceive frequency multiples as equal steps).

To calculate a musical interval in an equal tempered scale in which the octave is divided into X equal parts, one can use the following equation.

\[
\text{fitness} = \left| \frac{\text{logarithmically equally tuned}}{\text{measured}} - 1 \right|
\]

<table>
<thead>
<tr>
<th>Note</th>
<th>Barung</th>
<th>Demung</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.003638697528</td>
<td>0.00304736031</td>
</tr>
<tr>
<td>2</td>
<td>0.00589800023</td>
<td>0.004785719045</td>
</tr>
<tr>
<td>5</td>
<td>0.01007135591</td>
<td>0.006660910896</td>
</tr>
<tr>
<td>6</td>
<td>0.005904965587</td>
<td>0.002640151159</td>
</tr>
<tr>
<td>1</td>
<td>0.006196377502</td>
<td>0.002868068834</td>
</tr>
<tr>
<td>Average</td>
<td>0.006341879351</td>
<td>0.004000442049</td>
</tr>
</tbody>
</table>

Earlier in this paper we discussed the method of counting beats to identify frequencies. We settled on particular frequencies when no beats could be counted, which was limited in its precision by the fact that our samples of the sarons we in 2 second segments, but nonetheless resulted in precision to about 0.25 hertz. The difference between the measured frequencies of the Slendro instruments, and the frequencies we would predict from a 5 tone equal temperament scale, are about 1 to 1.5 hertz for the lower frequencies to 4.7 to 6.7 hertz for the higher frequencies (these numbers are derived from the average fitness numbers above). These numbers are several times larger than 0.25, meaning there would be a very audible difference if these contrasting frequencies were played concurrently, though such a comparison is probably not meaningful.

JUST INTONATION

At the very beginning of the prior section of this paper, we noted that the notes in the Slendro scale are evenly spaced across the octave. That fact naturally leads us to assess how close the notes are to begin perfectly equal in their distribution across the octave, as we did in the prior section. We should however, recognize that being approximately equal is a property present even in scales that are not equal tempered.

Just intonation is another system of tuning, one that has not only appeared in virtually every music culture in the world, but has also been pervasive for most of the worlds music history. A just scale, is one that contains intervals that can be expressed as rational numbers.

Assessing whether or not the measured frequencies fit with a just intonation scale is more difficult, because unless while 5 tone equal intonation is one particular scale, 5 tone just intonation is a class of infinite scales. To narrow our list of possible just scales to consider, lets identify some common properties of just scales actually used in the worlds music cultures.

1. Almost every just interval, is a simple rational number, meaning, the values b and c in b / c, contain very few prime factors, and contain prime factors that are low in value. 3 / 2 is perhaps the
most abundant musical interval in the world aside from the octave, and it contains just one prime factor
in the numerator\[xii\]. Complicated rational numbers, like 128/81, are far rarer.

2. Just scales usually contain prime factors that are low. In my studies, I have never heard of a just
interval containing a prime factor greater than 13. Prime numbers 3, 5, and 7 are present in many of
the worlds just scales, with 3 being in virtually all of them, 5 being common, and 7 being less common.
11 and 13 are virtually absent.

3. Just scales often demonstrate monophony, meaning that for every just interval in the scale, its
reciprocal also appears in the scale. If 5 / 4 is in a monophonic scale, then 8 / 5 is also in the scale\[xiii\].

4. Just scales usually contain intervals that are consonant with each other. Consonance here,
meaning that the interval between them is also a simple rational number containing low factors. For
example, 9/8 and 3/2 are consonant to each other, because 9 / 4 divided by 3 / 2 is the low prime value
of 3 / 2, but 20 / 9 divided by 3 / 2 is 40 / 27, a containing higher value primes, as well as more prime
factors.

With the constraints mentioned above, and with the intention of finding intervals relatively close to the
measured intervals we can derive the following just scale..

\[
\begin{align*}
1/1 \\
8/7 \\
21/16 \\
32/21 \\
7/4 \\
2/1
\end{align*}
\]

Comparing this scale to the measured intervals in absolute and relative terms is listed below..

<table>
<thead>
<tr>
<th>Note</th>
<th>Barung</th>
<th>Demung</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.001453765491</td>
<td>0.008173996176</td>
</tr>
<tr>
<td>2</td>
<td>0.0005901311907</td>
<td>0.0005280888646</td>
</tr>
<tr>
<td>3</td>
<td>0.00470686368</td>
<td>0.001314531549</td>
</tr>
<tr>
<td>5</td>
<td>0.007898679014</td>
<td>0.002458344715</td>
</tr>
<tr>
<td>6</td>
<td>0.06196377502</td>
<td>0.002868068834</td>
</tr>
<tr>
<td>1</td>
<td>0.002747401153</td>
<td>0.003068606027</td>
</tr>
</tbody>
</table>

Average 0.002747401153 0.003068606027

In the equal intonation section, we assessed how perceptibly different these predicted tones would be
from the measured tones is play concurrently. For the absolute measurement we can see a hertz
difference of 0.75 hertz to 4 hertz, which is still perceptibly different.

**COMPARISON BETWEEN JUST INTONATION AND EQUAL TEMPERAMENT MODELS**

Within the absolute interval comparison, the Just Intonation model is closer to the measured intervals
with about twice the fitness (values half of the equal temperament fitness). The tones predicted by our
just intonation model are not only closer to the measured intervals, but closer to them than they are to
the equal tempered values.

**ANALYTIC WEAKNESSES AND CONSIDERATIONS**

**CONCERNS ABOUT OVERFITTING THE DATA**

Because the just scale was devised deliberately to fit the data we should proceed with skepticism. We could devise a scale, that simply produces the exact intervals we measured, and our model would fit the data perfectly. Such a model, tho fitting perfectly, would not be meaningful, because there is no underlying theory behind the model. In this paper I have constrained myself only to doing analysis which I believe would have meaningful implications about the culture and practice of Gamelan tuning.

**SCOPE OF THE GAMELAN ORCHESTRA ANALYZED**

In this paper we looked at measured frequencies from only two instruments spanning two octaves. Both in terms of instrumentation, and the range of octaves, these two instruments represent a minority of the Gamelan Orchestra. Perhaps if we analyzed higher and lower instruments we would find them to break with our tuning model\[xiv\].

**POSSIBLE INSTRUMENT 'PAIRING'**

Beating is probably a deliberate feature of Gamelan tuning. Within the Gamelan Orchestra there are many Sarons, some even sharing the same octave. It has been suggested that Gamelan instruments within the same orchestra are deliberately tuned slightly differently than each other to produce beating. If this is true, then the implications of this for our analysis, would be some Sarons are tuned slightly higher and others slightly lower, and some of the measured error in our fitness assessment would correspond to opposite error in a different Gamelan instrument that I did not record.

**UNIQUE TUNING PER GAMELAN ORCHESTRA**

As we discussed in the brief introduction to Gamelan section, each Gamelan orchestra is tuned differently. We have seen that the Just intonation model is a better explanation of the tuning system apparent in the Javanese Gamelan orchestra at Arizona State University, but how do we account for variance between orchestras? There are two factors to consider. The first, is that perhaps Gamelan orchestras are tuned to different Just intonation scales, as it is unlikely that Gamelan orchestras would be tuned to fundamentally different tuning theories, and more likely that they are tuned to different varieties of the same tuning theory. Second, is that because Gamelan is not a harmonic style of music, and because the resulting beating from imprecise tuning is somewhat desirable, Gamelan instrument makers might aim very precisely for a frequency greatly variant from the ideal just interval. Different Gamelan orchestras could be tuned to the same just intonation scale, but all subsequently be incompatible, due to the differently accepted variance from a target just interval.

**OTHER LITERATURE**

Edward C. Carterette and Roger A. Kendall wrote 'On the Tuning and Stretched Octave of Javanese Gamelans'. In their paper, they primarily focus on the phenomenon of octave stretching, and they do little to examine the exact placement of the notes in the Slendro scale. Not only does this disappoint me, but they also neglect to share the recorded frequencies for the Gamelan ensembles they looked at. They do however, share the average the intervals recorded from the Slendro scale, and list them in the paper. We should note however, that the purpose of averaging intervals isn't without large problems.\[xv\] We shall do a cursory analysis of those average values, which I will list below\[xvi\]

<table>
<thead>
<tr>
<th>Note</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.142742</td>
</tr>
<tr>
<td>3</td>
<td>1.312666</td>
</tr>
</tbody>
</table>
Carterette and Kendalls intervals make for a peculiar comparison with the intervals recorded from Arizona State Universitys Javanese Orchestra. The notes 2 and 3 are very similar, and the notes 5 and 7 are not similar.

Carterette and Kendalls recorded interval for the note 2 and note 3, incredibly, are almost exactly equal to the just intervals 8 / 7, and 21/16.

\[\frac{|8/7 - 1.142857|}{1.142857} - 1 = 0.00100785 \]

\[\frac{|21/16 - 1.3125|}{1.3125} - 1 = 0.00012586 \]

This is a level of exactness that would exceed anyones anyone standard for precision in tuning. For instruments in the middle range of human hearing, it would take a full 20 seconds for a beat duration between 8/7 and Carterette and Kendalls recorded average for Slendro note 2.

Kendall and Cartettes recorded interval for note 5 and note 6 deviates from what we recorded from Arizona State Universities Javanese orchestra. Note 5 roughly corresponds to 3 / 2. To my knowledge, Kendall and Carterettes recorded value for note 6 has no strong similarity to a simple just interval, it sits roughly equally between just intervals 7 / 4 and 12 / 7, and is closer to the equal tempered note . It could be that Slendro notes 6 in Gamelan orchestras, are always tuned in equal proportion to either 7 / 4 or 12 / 7.

A just intonation model for Kendall and Carterettes recorded intervals fit far better than an equal intonation model, especially with regards to notes 2, 3, and 5. Carterette and Kendalls average recorded Slendro note 6 (2 to the 4 / 5 power ) is closer to the equal intonation value, but not to an exceptional extent, and like we mentioned above different just intonation values for note 6 in different orchestras could result in Carterette and Kendalls average.

CONCLUSION

Tho the methods in this paper are limited, we found that modelling the tuning of ASUs Javanese Gamelan orchestra as a just intonation scale to be superior to modeling as a 5 tone equal temperament. By comparing our limited findings to known information of other Gamelan Ensembles, we have no reason to doubt just intonation models would also better explain the tuning of other Gamelan Ensembles. The great variance and ambiguity in Gamelan tuning could be explained by different selections of just intonation intervals, intentional beating effects, and indifference to tuning in a style of music in which harmony is not essential.

ENDNOTES

[i] Metallophones being instruments with metal bars, and Gongs being instruments with metal disks.

[ii] The upper 1 being an octave above the lower 1.

[iii] I cannot say with perfect confidence why 4 is omitted. The Pelog scale is notated 1 through 7, including 4. It might be that Gamelan musicians see a parallel between the notes of the Slendro scale,
with the 1, 2, 3, 5 and 6 of the Pelog.

[iv] When two tones of the same frequency are in phase with each other, their amplitudes at any time t are the same, and therefore combine when sounded together. Putting aside how sound reflects in a physical environment, a listener hearing two tones with the same frequency and amplitude perceives the tone twice as loud than if he heard one tone at the frequency. When two tones of the same frequency are out of phase, their amplitudes at any time t are opposite, and therefore cancel out when sounded together. A listener would hear silence if two tones of the same frequency but out of phase were sounded.

When two tones are not of the same frequency, they oscillate between being in phase with each other, and out of phase with each other. For many pairs of tones, this oscillation happens on a very short time scale. For example, 100 hertz and 150 hertz complete an oscillation between being in and out of phase every 0.02 seconds (50 hertz).

[v] 0.5 hertz divided by two, since a beat would still be obvious to the listener if it lasted less than a full cycle, and more than half a cycle.

[vi] A theoretical harmonic is a whole number multiple of a base frequency. The harmonics of 100 hertz are 200, 300, 400, 500 etc hertz. Bodies that are resonant with a hertz B, typically are also resonant with harmonics of B. When musical instruments sound a note B, many other frequencies also sound, many of which are harmonics of B. In actual practice however, a frequency B is sounded with frequencies that are not exactly equal to its theoretical harmonics, but are in fact slightly higher than its theoretical harmonics. This is due to the fact that the frequency produced by a resonating body is a function of its tension, and the tension on a body is a function of its shape, which is in flux during vibration. So for example, when a string vibrates at hertz B, it becomes slightly longer, which increases the tension on the string, which biases the frequency higher for all concurrently sounding frequencies higher than B (such as the harmonics). In practice, a body resonating at 100 hertz, would more likely produce near harmonics such as 202, 304, and 408 hertz. The higher the harmonic, the more biased it is, since it is affected by the tension increasing effect of more sub frequencies.

[vii] We could also compare the intervals in relative terms, meaning each interval is a comparison between one note and the prior one in the scale. I have done a preliminary relative analysis, but I have chosen to omit it from this paper. A relative analysis is more complicated, less meaningful, and would not reach conclusions independent of an absolute analysis. Most every tuning system in practice, is devised relative to a base frequency, and in the literature of musical tunings absolute comparisons are the norm.

[viii] While this is something we are assuming, it is largely irrelevant as to which tone we assume is the base tone.

[ix] Meaning, 100 to 200 hertz is an octave, a multiple of 2, and a difference of 100 hertz (200 - 100 = 100), while 800 to 1600 hertz is also an octave, also a multiple of two, but a difference of 800 hertz. People identify frequency steps by their multiple, not their integer frequency difference.
Frequency precision is valuable to music in which harmony is a component, and Gamelan is not a music culture in which harmony plays an important role.

A rational number, being a number that can be expressed as $b / c$, for integer values of $b$ and $c$). 1.25, is a just interval, because it is equal with $5 / 4$, and 5 and 4 are integers.

That being 3, we should exclude 2 from our consideration for two reasons. First, because listeners identify octaves as the same note. 9/8 is an octave below 9/4, so the presence of an extra 2 in the denominator is not a component of 9/8’s identity as a musical interval. Second, because for every tone to be in a scale, it must be between 1 and 2, so we pick the octave of each interval that is between 1 and 2 (3/2, rather than 3/1 or 3/4).

That being 3, we should exclude 2 from our consideration for two reasons. First, because listeners identify octaves as the same note. 9/8 is an octave below 9/4, so the presence of an extra 2 in the denominator is not a component of 9/8’s identity as a musical interval. Second, because for every tone to be in a scale, it must be between 1 and 2, so we pick the octave of each interval that is between 1 and 2 (3/2, rather than 3/1 or 3/4).

However, we have no reason to expect that.

To what extent this is valuable, is dubious, for the reasons mentioned in the section of this paper 'Uniquely tuning per Gamelan Orchestra'. If two Gamelan orchestras were tuned perfectly to two different just intonation scale, their average would be imperfectly tuned to both. Also, if two Gamelan orchestras were tuned with a random degree of imprecision to the same just intonation scale, then we could only devise the just intonation scale from a statistically sufficient number of orchestras. That hey are tuned with non random imprecision, to a variety of just intonation scales, is both more plausible, and more confusing to the purpose of averaging many Gamelan orchestras.

Carterette and Kendall list the intervals in cents, which can be mathematically reformulated in the decimal expression we have been using.