INTRODUCTION

Despite the large progresses in theoretical and experimental physics there are still a couple of phenomena which cannot be described by modern and well-established theories. Such phenomena require an extension of well-known physical models. An example of such an extension is the Supersymmetric Standard Model (Fayet 1975). Since the Standard model doesn’t include gravitational interactions, also Quantum Gravity models were proposed (Sundance 2007). A recent extension of the Standard model is the Topological Dipole Field Theory (TDFT) which assumes that gauge bosons couple to an intrinsic dipole moment (Linker 2015). This theory adds a 2-form field \( B' \) to the ordinary 2-form gauge field strength \( F^A = dA + g A \wedge A \) where \( B' \) is an additional degree of freedom, \( A \) is the gauge connection and \( g \) is the gauge coupling constant. The field \( B' \) is an observable of a Witten-type topological quantum field theory. TDFT is originally based on Čech cohomology and can be formulated with a Lagrangian density that is independent on the choice of the topological bases that generate the Čech cohomology. The Lagrangian density of TDFT depends on the 2-form fields \( B' \) and \( W \) and also on a Lagrange multiplier \( \lambda \) where it is integrated over all possible real-valued fields \( B', W \) and \( \lambda \). The 4-form topological Lagrangian density has the form:

\[
L = \text{tr}(b B' \wedge W + \lambda W \wedge W).
\]  

(1)

Here, \( b \) is the coupling constant for the coupling to the topological dipole. The Lagrangian (1) generates a Lorentz-invariant and real-valued action. If the action is dimensionless and lengths or times have dimension 1 the 2-form fields \( B' \) and \( W \) must have dimension -2, while \( b \) and \( \lambda \) are dimensionless.

An example of an unsolved problem in physics is the Baryon asymmetry (Sakharov 1991). Baryon
asymmetry means that number of baryons is higher than the number of antibaryons in the universe.

Another unsolved problem in physics is the origin of dark energy (Peebles et al. 2003). The fluctuations in dark energy are much stronger than the Standard model of particle physics is predicting. These phenomena can be described by TDF more precisely since there are additional terms involved that modify the dynamics of some scattering processes. Moreover, phenomena related with dark energy and the creation of particles and antiparticles depends strongly on the dynamics of gauge bosons.

In this research paper it is shown how to calculate scattering processes that arise from these additional terms of TDF. From the structure of the Lagrangian density Feynman rules for scattering with intrinsic topological dipoles are constructed. Also expectation values of topological dipole correlations are computed. After the treatment of basic Feynman rules, a transition probability per time is computed for a simple scattering process. This scattering process is a modified propagation of a gauge boson within a short time. After the calculation of the transition amplitude, the computational results are discussed with respect to baryon asymmetry and dark energy.

THEORY

The theory is based on the topological action (1) on a spacetime manifold $M$ which can be written in coordinate basis that is denoted by Greek indices as follows:

$$S_{\text{top}} = \frac{1}{4} \int_M d^4x \varepsilon^{\mu\nu\rho\sigma} (b^a B_{\mu\nu}^a W_\rho^a + \lambda W_{\mu\nu}^a W_\rho^a).$$

Here, the Latin indices (here: $\alpha$) are running over all Lie group generators $T_\alpha$ of the gauge theory which satisfy $\text{tr}(T_\alpha T_\beta) = \delta_{\alpha\beta}.$

FEYNMAN RULES FOR TDF

For the Standard model of particle physics Feynman rules are well-known. Feynman rules for Standard model propagators and vertices are in TDF exactly the same as in the ordinary Standard model. The only difference between the TDF and the Standard model is that there are occurring the interaction terms $\frac{1}{2} \int_M d^4x B_{\mu\nu}^a F_{\mu\nu}^a$ and $\frac{1}{4} \int_M d^4x B_{\mu\nu}^a B_{\rho\sigma}^a$ when the generalized 2-form field strength $F = F_0 + B$ is expanded around $F_0$ in the Lagrangian density. When these interaction terms are expanded out in the path integral one obtains additional self-interactions in the gauge bosons. More precisely, one obtains for all possible interactions with topological dipoles the expression:

$$\exp[iS_{\text{int}}] = \sum_{I,J=0}^{\infty} \frac{1}{I!J!} \left( \frac{1}{2} \int_M d^4x B_{\mu\nu}^a F_{\mu\nu}^a \right)^I \left( \frac{1}{4} \int_M d^4x B_{\mu\nu}^a B_{\rho\sigma}^a \right)^J. \quad (3)$$

From (3) it is easy to see that there is a product of $I + 2J$ fields $B^I$ in the term indexed with $(I,J).$ If the gauge theory is abelian then the term indexed with $(I,J)$ describes $I$ self-interacting gauge bosons. In the nonabelian case, the factor $\left( \frac{1}{2} \int_M d^4x B_{\mu\nu}^a F_{\mu\nu}^a \right)^I$ can be expanded in powers of $\Pi = \int_M \text{tr}(B^I \wedge A \wedge A).$ May be $q$ the power of $\Pi$ occurring in the Taylor series expansion. Then the term indexed with $(I,J)$ has $I + q$ self-interacting gauge bosons. A Feynman diagram which describes a $^I$N-boson self-interaction is denoted as a rectangle with $^I$N legs (which illustrating incoming
and outgoing bosons) that are attached to this rectangle. Inside the rectangle two numbers $q \cdot r$ separated with comma are written. The number $q$ is the power of $B'$ while $r$ is the power of the contribution $\int d^4x \, tr(B' \wedge \bar{B}')$. Consequently, there has to be computed the expectation value of a product of $2r + n - q$ tensor fields $B'$. The following example shows a Feynman diagram for a 4-boson interaction.

![Feynman diagram](image)

Figure 1: 4-boson interaction; Bosons are denoted by straight lines

For usual, there are written ingoing and outgoing momenta and polarizations on the legs of the rectangle. Moreover, the product of fields $B'$ which has to be computed is written next to the rectangle. The product $A^{a_1 \ldots a_N}_{a_2 \ldots a_N}$ of $N$ fields $B'$ is defined as

$$A^{a_1 \ldots a_N}_{a_2 \ldots a_N} = \langle \prod_{j=1}^{N} B'^{a_j b_j} \rangle,$$

where the averaging $\langle \cdots \rangle$ denotes the weighted average with weighting factor $\exp(i S_{\text{top}})$. An easy calculation shows that (4) can be written as:

$$A^{a_1 \ldots a_N}_{a_2 \ldots a_N} = \int d[\lambda] \left( \frac{ib}{2} \right)^{-N} \prod_{j=1}^{N} \theta_{a_j b_j} \omega_{a_j \mu} \omega_{b_j \nu} \exp \left( \frac{i}{2} \int d^4x \lambda^{\rho} \rho^{\mu \nu} W_{a \rho} W_{b \nu} \right) \big|_{W=0}. \quad (5)$$

If (5) is further evaluated one observes that the exponent of $\lambda$ in the integrand of (5) increases with $N$. Integration over $\lambda$ and normalizing (in this research paper it is assumed that the path integral normalization factor is included in the functional integration measure) yields powers of an infinite quantity $\eta \rightarrow \infty$. In nonvanishing terms $A^{a_1 \ldots a_N}_{a_2 \ldots a_N}$ this infinite quantity is of order $\eta^{2N}$ and can easily be absorbed in the bare coupling constant $b$ by setting $\sqrt{\eta} = b'$. The coupling constant $b'$ is (when other UV divergences are absorbed) a physical constant. The minimum number of factors $N$ where (5) is nonvanishing is $N = 4$ since it holds $\int d[\lambda] \lambda(x) \lambda(y) = \frac{\delta_{xy}^2}{2}$ and $\int d[\lambda] \lambda(x) = 0$.

MODIFIED PROPAGATION OF A BOSON

Considering a vector boson with initial energy-momentum state $k^{\mu}$ and initial polarization $\chi^{a}$, the vector boson goes to the final energy-momentum state $k'^{\mu}$ and final polarization $\chi'^{a}$. Then the
probability density that this boson is involved in this reaction within a time \( T \) and within a volume \( V \) denoted by \( w(k_\mu, k'_\mu; x_{\nu,1}, x_{\nu,2}) \) in the Lorentz-invariant phase space volume \( (2\pi)^4 \Delta x_{\nu} \) has the form:

\[
w(k_\mu, k'_\mu; x_{\nu,1}, x_{\nu,2}) = \frac{4E^2}{4E^2 T^2 (2\pi)^3} \Omega \]

(6)

Here, \( E \) is the kinetic energy of the incoming boson, \( E' \) the kinetic energy of the outgoing boson and \( \Omega \) the expectation value of the T-Matrix associated with the 2-boson self-interaction. From (3) it can be observed that the term \((i = 2, j = 1)\) is the only term with nonvanishing topological expectation value \( \Lambda^{x_{\nu,1} x_{\nu,2}} \) and without boson loop contributions which describes a 2-boson interaction. The UV divergent Loop contributions which arise in higher exponents \( i \) can be absorbed by renormalization. For simplicity, only the term \((i = 2, j = 1)\) is considered. Transforming this term into \( k\)-space one obtains:

\[
\Omega = \frac{1}{16} \int k_{\mu} x_{\nu,1} k'_{\mu} x_{\nu,2} \Lambda^{x_{\nu,1} x_{\nu,2}} \Delta x_{\nu} \text{sinc}(k - k') g_{xx} \Delta x_{\mu}.
\]

(7)

In equation (7) the expression \( \Delta x_{\nu} \) denotes the observation interval in spacetime direction \( K \) (with \( \Delta x_{\nu} = T \) and \( V = \Delta x_{\nu} \Delta x_{\mu} \)) and a contraction in \( \Lambda^{x_{\nu,1} x_{\nu,2} x_{\nu,3}} \) due to the factor with \( j = 1 \) was performed. Also fluctuations in energy and momentum are included in equation (7). Therefore the transition probability per unit time (6) also allows processes with nonconservation of energy and nonconservation of momentum. For large observation time the sinc-function can be approximated as a delta distribution and energy and momentum is conserved in transitions. The transition rate is proportional to \( b^{-3} \) and is assumed to be small; hence, a small deviation from ordinary Standard model is included by adding TDFT transition probabilities. If the boson polarizations in (6) does not matter it can be averaged over all possible polarizations. In the limit of very large momentum absolute values \( k, k' \) the transition probability (6) decreases as \( w \propto |k k'|/|k - k'|^5 \). It makes sense that the transition probability decreases rapidly with momentum excess \( \xi_k := |k - k'| \)

CONCLUSIONS

The transition amplitudes given by the equations (6) and (7) are a model for boson density fluctuations. Bosons that undergo stochastic noise in small time scales can explain why dark energy fluctuations are much higher than the Standard model of particle physics predict. It makes sense that the highest amount of dark energy was present a few Planck times after the Big Bang since fluctuations predicted by TDFT were very significant. Baryon asymmetry arises from the high fluctuations in energy and momentum during this time period. The high-energy regions tend more to particle-antiparticle creation than the low-energy regions.

REFERENCES


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