the
WINNOWER

PHYSICS

E-gravity theory

PATRICK LINKER

A couple of quantum gravity theories were proposed to make theoretical predictions about the behavior of gravity. The most recent approach to quantum gravity, called E-theory, is proposed mathematical, but there is not formulated much about what dynamics of gravity this theory proposes. This research paper treats the main results of the application of E-theory to General relativity involving conservation laws and scattering of particles in presence of gravity. Also the low-energy limit of this theory is shown.

CORRESPONDENCE:
patrick.linker@t-online.de

DATE RECEIVED:
February 02, 2016

DOI:
10.15200/winn.145441.18359

ARCHIVED:
February 02, 2016

KEYWORDS:
quantum gravity, general relativity, quantum fluctuations, quantum field theory

CITATION:
Patrick Linker, E-gravity theory, *The Winnower* 3:e145441.18359 , 2016 , DOI: 10.15200/winn.145441.18359

1. INTRODUCTION

Quantum gravity theories focus on the concept of a quantization of General Relativity which has no UV-divergences. Such theories become relevant a few Planck lengths after the Big Bang due to the high energy densities that were present in this time period. Many approaches of quantum gravity were proposed in research literature. The most popular approach to renormalizable quantum gravity is String theory, which assumes that there are additional dimensions in the universe. Moreover, it uses the concept of supersymmetry. Elementary particles are not pointlike; they have a form of a string or membrane such that the momentum uncertainty remains finite independent on the backgrounds in which these objects are. However, these concepts are still not validated experimentally.

Recently, a new approach of quantum gravity was developed (Linker 2016). This approach uses a semigroup called “E-semigroup” to discretize the spacetime structure. By the introduction of a generalized curvature measure, the action of General Relativity can be rewritten. The key observation to do this is that the chain complex in differential geometry of smooth manifolds (the deRham complex on Riemannian manifolds or smooth fiber bundles) is losing the exactness if the space has nonvanishing curvature. Causal Dynamical Triangulation (CDT) has similarities with this theory (Ambjørn 2013). With CDT the spacetime structure is also formulated discrete (Loll 1998). Here, the key ingredient is the Regge calculus that provides a proper discretization of the spacetime (Immirzi 1997). Despite the large efforts of quantum gravity theories in the last decades it has to be made clear which quantum gravity theories predict regarding gravity on microscopic scales.

This research paper shows how E-theory can be applied to a scalar boson which underlies gravitational effects in 4-dimensional spacetime. The low-energy limit, conservation laws and the perturbative calculation of scattering amplitudes are shown. With these considerations the general procedure of an application of E-theory to gravitational physics called “E-gravity” is made clear. Similarities between this quantum gravity theory and Topological Dipole Field Theory (Linker 2015) are also shown in this research paper. Topological Dipole Field Theory (TDFT) is a model that describes a modification of the dynamics of gauge bosons which implies distinct behavior of quantum fluctuations.

2. THEORY

The basic concept of E-gravity ist the E-semigroup S. Assuming that the 4-dimensional spacetime can be triangulated by simplices. Then, this E-semigroup has the generators $s_0, \dots, s_4 \in S$ for the 4-dimensional spacetime. Here, the reason for the choice of exactly five generators is that 4-dimensional simplices have five corners. When multiplying some generators one has elements

$s_{ij} \in S, 0 \leq i \leq 4, 0 \leq j \leq 4, i \neq j$ that represent lines between corners generated by s_i, s_j in a geometrical sense. Similary, triangles and its higher-dimensional generalizations can be realized by multiplying the corresponding generators (Property (i) of E-semigroups). Also, a simplex might not be well-defined (Property (ii) of E-semigroups). When a simplex is build up on two identical corners, it is not well-defined (Property (iii) of E-semigroups). Clearly, arbitrary simplex elements, where corners are labeled by numbers, are not changing when the labeling numbers are interchanged (Property (iv) of E-semigroups).

Curvature arises from simplices (or more general: subspaces of topological spaces) that are not well-defined; this leads to a loss of exactness in a chain complex. A very important function that is used in E-gravity theory is the indicator function I. This indicator function has the value 0 if it is applied on an empty set and 1 otherwise. It holds for the generalized curvature 2-form on a simplex generated by the generators $s_0, s_1, \dots, s_4$ the formula

$$\Omega = \delta^2 I(s_0 s_1 \dots s_4), \qquad (1)$$

where δ is the corresponding coboundary operator. May be Π the set of all simplices, then the action of quantum General Relativity reads:

$$S_{GR} = \nu \sum_{z \in \Pi} \Omega|_z. \qquad (2)$$

Here, ν is a coupling constant and the operation $|_z$ denotes the evaluation at the simplex z. It is easy to prove the low-energy limit of the action (2). Let be $Z \subset \Pi$ and $\mathrm{vol}(Z) \equiv \sqrt{-g} d^4x$ with metric tensor $g_{\mu\nu}$ and ist determinant g. Then the expression $R = \sum_{z \in \Pi} \Omega|_z / \mathrm{vol}(Z)$ denotes the average curvature measure; this measure can be identified with the Ricci scalar. Also the Ricci scalar can be regarded as proportional to the direction-averaged Riemann curvature tensor weighted with metric tensor, i.e. $R = R_{\mu\nu\alpha\beta} g^{\mu\alpha} g^{\nu\beta}$.

An example of a quantum gravity theory with matter sources is the gravitational ϕ^4-theory. The action of the nongravitational ϕ^4-theory has the form:

$$S_{mat} = \int (d\phi^* \wedge * d\phi + g_0 (\phi^* \phi)^2 * 1). \qquad (3)$$

In this action, g_0 is a coupling constant and ϕ is the 0-form scalar field. It is easy to include gravity in the action (3). Continuum integrals have to be replaced by a sum over a discrete set of simplices (similar to the discretization of integrals in finite element methods). It is obvious to set:

$$\int g_0 (\phi^* \phi)^2 * 1 = \sum_{z \in \Pi} [\phi^2 (s_0 \dots s_4) \phi^{*2} (s_0 \dots s_4)]|_z \, g_0. \qquad (4)$$

To rewrite the kinetic energy term of (3) one has to transform ordinary directional derivatives to derivatives in direction of outward normal vectors in simplices. From linear algebra it is known that a linear combination of field quantities on a simplex surface element (dependent on the triangulation that is used) yields a directional derivative. By using the relation $\partial_\mu \phi = \sum_{j=0}^4 \kappa_{\mu j} \phi(s_0 \dots \widehat{s_j} \dots s_4)$, where the superscript $\widehat{\cdots}$ denotes that the factor is omitted and $\kappa_{\mu j} \in \mathbb{R}$ are the transformation factors that depend on the triangulation, one obtains:

$$\int d\phi^* \wedge * \, d\phi = \sum_{z\in\Pi}\left[\sum_{j,k=0}^{4} \kappa_{\mu j}\kappa_k^{\mu}\phi^*\left(s_0 \dots \hat{s_j} \dots s_4\right)\phi\left(s_0 \dots \hat{s_k} \dots s_4\right)\right]\Big|_z . \qquad (5)$$

Because of the fact $\phi(0) = 0$ (field values are vanishing on empty sets) one can also write any quantum field as $\phi(x)|_z = \phi^{\#}(x)|_z I(x)|_z$ with $x \in S, z \in \Pi$, where the superscript $\dots^{\#}$ denotes that this quantum field exists independent on the equaliser set (it exists even on empty sets). With this decomposition, one has the information about gravity included only in the indicator function. Finally, the action (3) in presence of gravity can be written as:

$$S_{mat} = \sum_{z\in\Pi}\left[\sum_{j,k=0}^{4} \kappa_{\mu j}\kappa_k^{\mu}\phi^{\#*}\left(s_0 \dots \hat{s_j} \dots s_4\right)\phi^{\#}\left(s_0 \dots \hat{s_k} \dots s_4\right)\right]\Big|_z + S_{grav}. \qquad (6)$$

The term S_{grav} contains all information about the coupling of matter with the gravitational field, i.e. products of indicator functions with matter field. It is clear that the total action of E-gravity with spin-0 boson has the form: $S_{tot} = S_{GR} + S_{mat}$. Only the term S_{grav} includes boson self-interactions and gravitational interactions.

May be Λ the set of all equalizers that are possible in the set S, one has the following path integral for E-gravity:

$$Z = \sum_{\Lambda}\int d[\phi^{\#}]e^{iS_{tot}}. \qquad (7)$$

The Feynman propagator (7) is a path integral over a set of inhomogenous spacetime strucutres. Local translations in space and time yield different structures of spacetime. This implies a nonconservation of energy and nonconservation of momentum by Noether's theorem. Spin-angular-momentum is also not conserved by Noether's theorem, because local rotation of simplices also generate a different structure of spacetime in general. Only on length and time scales large in comparison with Planck length and Planck time, energy-momentum and spin-angular-momentum is covariantly conserved; by averaging up inhomogenities and anisotropies of spacetime, the effect of inhomogenity and anisotropy becomes smaller such that it can be assumed to be conserved in a covariant way.

Scattering amplitudes from E-gravity can be computed by expanding the factor $e^{iS_{grav}}$ of the propagator (7) in Taylor series. One can perform the following decompositions:

$$S_{grav} = S_I + S_{II}, \qquad (8a)$$

$$S_I = \sum_{z\in\Pi}\left[\left[\sum_{j,k=0}^{4} \kappa_{\mu j}\kappa_k^{\mu}\phi^*\left(s_0 \dots \hat{s_j} \dots s_4\right)\phi\left(s_0 \dots \hat{s_k} \dots s_4\right)\right]\Big|_z - \sum_{j,k=0}^{4} \kappa_{\mu j}\kappa_k^{\mu}\phi^{\#*}\left(s_0 \dots \hat{s_j} \dots s_4\right)\phi^{\#}\left(s_0 \dots \hat{s_k} \dots s_4\right)\right]\Big|_z, \qquad (8b)$$

$$S_{II} = \sum_{z\in\Pi}[\phi^2(s_0 \dots s_4)\phi^{*2}(s_0 \dots s_4)]|_z g_0. \qquad (8c)$$

While S_I is a pure gravity-boson coupling term, S_{II} describes ordinary boson self-interactions as well as gravitational effects. Since the gravity field is governed by a non-gaussian distribution, nonvanishing higher stochastic moments are possible. Therefore, it is difficult to draw gravitational interactions in Feynman diagrams; these interactions have to be computed non-perturbatively. However, a perturbative expansion with gravitational effects contains also a vertex with only one ingoing boson and only one outgoing boson due to interactions generated by S_I.

Such a type of interaction vertex also occurs in TDFT. In the case of TDFT, an interaction generated by a single gauge boson correspond to a quantum fluctuation occuring while the gauge boson propagates. Also in E-gravity, a gravitational self-interaction of a boson (or more general: a particle) is an interaction very similar to a quantum fluctuation in TDFT. Fluctuations in particle energy and particle momentum can be induced by TDFT interactions as well as by gravitational interactions.

3. CONCLUSIONS

E-gravity is a quantum field theory, where basic integrals of motion like energy are not conserved in general. Only in the low-energy regime these integrals of motions can exist. Scattering processes predicted by E-gravity contain also self-interactions of particles due to gravity. This modifies the behavior of quantum fluctuations in a similar way as TDFT does. Gravitational modifications of quantum fluctuations apply on all elementary particles and gauge bosons and not only on gauge bosons.

REFERENCES

Linker, P. “Foundations of E-Theory”. *The Winnower*, 2016, 3:e145350.06184. doi: 10.15200/winn.145350.06184. URL: https://www.thewinnower.com/papers/3339-foundations-of-e-theory

Ambjørn, J. et al. “Trees and spatial topology change in CDT”. *J. Phys. A: Math. Theor.* ,2013, 46: 315201. doi: 10.1088/1751-8113/46/31/315201

Loll, R ."Discrete Approaches to Quantum Gravity in Four Dimensions". *Living Reviews in Relativity*, 1998, 1: 13. doi:10.12942/lrr-1998-13. arXiv:gr-qc/9805049. Bibcode: 1998LRR.....1...13L

Immirzi, G. „Quantum Gravity and Regge calculus“. *Nucl.Phys.Proc.Suppl.* , 1997, 57: 65-72. doi: 10.1016/S0920-5632(97)00354-X

Linker, P. „Topological Dipole Field Theory“. *The Winnower*, 2015, 3:e144311.19292. DOI: 10.15200/winn.144311.19292