On the Cyclic Variations in Newton's Constant

BRENT JARVIS

1. Embry–Riddle Aeronautical University

Periodic oscillations in Newton's constant $G$ are contemporaneous with length of day data obtained from the International Earth Rotation and Reference System. Preliminary research has determined that the oscillatory period of $G$ is $\approx 5.9$ years ($5.899 \pm 0.062$ years), which is close to one–half the principle period of solar activity. It will be shown mathematically that the variations in $G$ are concomitant with gravitomagnetically induced torsion on the Earth's spin during the $\approx 5.9$ year period. The gravitomagnetic acceleration at the Earth's equator is in good agreement with experimental measurements and falsifiable predictions are given to test the torsion hypothesis.

INTRODUCTION

Measurements of $G$ oscillate between $6.672 \times 10^{-11}$ and $6.675 \times 10^{-11}$ N·(m/kg)$^2$ with a periodicity of $\approx 5.9$ years (a difference of $10^{-4}$%) [1, 2]. Scientists studying this anomaly have found that the variations can be predicted from length of day (LOD) data obtained from the International Earth Rotation and Reference System [3].

It was suggested by Anderson et. al. [2] that an actual increase in $G$ “should pull the Earth into a tighter ball with an increase in angular velocity and a shorter day due to conservation of angular momentum,”
which would contradict the G/LOD synchronicity in fig. 1. It can be shown mathematically, however, that this reasonable assumption is not entirely accurate.

The gravitational field produced by a massive spinning body in Einstein's general theory of relativity can be described by equations which have the same form as Maxwell's equations for electromagnetism (in the flat spacetime weak field limit). These equations are known as the gravitoelectromagnetic (GEM) equations [4], and the gravitomagnetic torsion $\xi$ (an effect of frame-dragging) is equivalent to half of the Lense–Thirring precession frequency [5, 6].

$$\xi = \frac{G}{2c^2} \frac{L - 3(L \cdot r) r}{r^3}$$

where $c$ is the velocity of light in a vacuum and $L$ is the body's angular momentum (according to Mashoon et al. [6] the denominator 2 "can be traced back to the spin–2 character of the gravitational field" as opposed to the spin–1 electromagnetic field in Maxwell's theory).

For brevity, let us assume the measurements of $G$ are taken at the Earth's equator so the dot product of $L$ and $r$ vanishes in Eq. 1. By approximating the Earth as a solid ball-shaped body of uniform density, $L$ can be expanded and we get

$$\xi = \frac{G2M_0\omega^2}{5c^2(2)^2} = \frac{G2M_0\omega}{5c^2\omega},$$

where $M_0$ is the Earth's rest mass, $\omega$ is its angular velocity, and $\omega$ is its diameter. Since LOD data indicates $\omega$ varies periodically [3], let us assume fig. 1 is accurate and $G$ varies with $\omega$. The velocity of light $c$ is a "rigid" constant, so $c^2$ can be used as the constant of proportionality and we get

$$c^2 = \frac{1}{5}G'M_0\omega;$$

where

$$G' = \frac{G + \Delta G}{\xi + \Delta \xi};$$

and $\omega = \frac{\omega + \Delta \omega}{\omega + \Delta \omega}$.

(we will see in a moment that the 2/5 ratio in Eq. 3 is linked to Euler's number). In order for $c^2$ to remain constant while $G$ and $\omega$ oscillate, $\xi$ and $\omega$ must also oscillate since $M_0$ is a conserved quantity. It can be deduced from Eqs. 3 and 4 that a decrease in the Earth's angular velocity $\omega$ (an increase in its LOD) would result in an increase in $G$, confirming the G/LOD synchronicity in fig. 1. It can also be deduced that an increase in $\xi$ would pull the Earth into a tighter ball, while an increase in $G$ would be contemporaneous with an increase in the Earth's oblateness.

ANALYSIS OF THE 2/5 RATIO

Reducing $c^2$ to $c$ in Eq. 3 we get

$$c = \sqrt{\frac{2}{5}} G'M_0\omega;$$

where

$$\sqrt{\frac{2}{5}} \approx 0.632$$

This ratio is commonly encountered in the realm of electrical engineering with the time constant $\tau$,

$$0.632 \approx (1 - \frac{1}{e^{\tau}}) \text{ when } t = \tau.$$ 

where $e$ is Euler's number, and

$$\tau = RC = \frac{L}{R} = \frac{1}{2\pi f_c} = \frac{1}{\omega e},$$

where $R$, $C$, and $L$ are the resistance, capacitance, and inductance of a series circuit respectively, and $f_c$ is the cutoff frequency ($2\pi f_c = \omega$). From the relationship

$$c^2 = \frac{1}{\epsilon_0\mu_0}$$

where $\epsilon_0$ and $\mu_0$ are the electric and magnetic constants respectively, an alternative version of Eq. 3 can be given as

$$1 = G'\omega \left(1 - \frac{1}{e^{\tau}}\right) M_0\epsilon_0\mu_0.$$
The Earth was approximated as a ball–shaped body of uniform density in Eq. 3, but Eq. 10 suggests that the rate of change in $G'$ and $\omega'$ are dependent upon the composition of $M_0$. If this is an accurate interpretation, $G'$ and $\omega'$ may change dualistically by the Euler factor $(1 - 1/e^t)$. Including the Lorentz factor from Einstein’s special theory of relativity, Eq. 10 can be given as

$$1 = G_2^2 \omega_2 M_0^2 \epsilon_3 \epsilon_4,$$

where $\epsilon_1$ and $\epsilon_2$ are the Euler factors and $\gamma$ is the Lorentz factor,

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}.$$

**CONCLUSION**

From Eq. 11, the gravitomagnetic acceleration $g(r)$ at the Earth’s equator is

$$g(r) = \frac{(G + \Delta G)}{r^2} \frac{M_0}{r^2} \left(\frac{(\omega + \Delta \omega)}{(\omega + \Delta \omega) + \epsilon_2 \epsilon_1^2 \gamma^2} \frac{r}{|r|}\right).$$

Even though the torsion $\xi$ is minute ($\approx 1.012 \times 10^{-14} \text{Hz} \text{at the equator}$) it has a hefty impact on gravity since it scales with $2c\epsilon$ ($\gamma \epsilon_1 \epsilon_2 = 2/5$ when $v << c$ so the torsion is essentially multiplied by $5c^{2}$ for the Earth’s rotation):

$$5c^{2} = (5)299,792,458 \approx 4.493775893684088 \times 10^{17} \text{m/s}.$$

Plugging and chugging for the gravitomagnetic acceleration proximal to the equator we get

$$g(r) \approx - \frac{5c^2(1.012 \times 10^{-14})}{(7.272205324 \times 10^{-3})(6,371,000)} \approx -9.796 \text{m/s}^2.$$

The CODATA recommended value (2014) for the standard acceleration of gravity is $9.806 (65) \text{m/s}^2$, which differs from the above approximation by $0.010 (65) \text{m/s}^2$.

As discussed in Eq. 8, the time constant $\tau$ in the Euler factors $\epsilon_1$ and $\epsilon_2$ is dependent upon a body’s resistance, and superconductors have zero resistivity [7]. This may explain why the gravitomagnetic acceleration measured by Tajmar et. al. (2006) [8] with spinning superconductors ($\approx 6,500 \text{rpm}$) was at least one hundred million trillion times greater than what was predicted with general relativity ($\approx 100 \text{millionths of the acceleration due to the Earth’s gravity}$). As stated by Tajmar, “We ran more than 250 experiments, improved the facility over 3 years and discussed the validity of the results for 8 months before making this announcement. Now we are certain about the measurements.”

It is hypothetically possible that the Sun’s gravitomagnetic field induces torsion on the Earth’s spin during the $\approx 5.9 \text{year G/LOD}$ period (close to one–half the principle period of its magnetic field reversals). If this is true, the Sun’s diameter $a$ and angular velocity $\omega$ may slightly fluctuate dualistically at nearly the same rate as the G/LOD variations (hysteresis would be expected due to the Earth’s inertia). It was shown by Holme R. and de Viron (2013) [3] that sudden changes in the Earth’s LOD are concomitant with jerks in its magnetic field, which may support this hypothesis. The anomalously low magnetic field of Venus may also be linked to its relatively low angular velocity ($\approx 243 \text{days in retrograde rotation}$).
fig. 2 An equation of time graph: Positive time values indicate an accurate clock ticking faster than a sundial and negative values indicate the opposite (in minutes). Annual variations in G should be detectable with torsion balance schemes. Taking into account the Earth’s obliquity (mauve dashed curve) and eccentricity (blue dash–dot curve), the difference between the annual variations in G (red curve) are hypothesized to be greatest on the dates marked by the green dots. G is predicted to be at maximum around 12 FEB and at minimum around 3 NOV (assuming the measurements are made proximal to the equator).

REFERENCES