The Baryonic Tully-Fisher Relationship: Empirical Evidence for a Baryonic Solution to the Galactic “Missing Mass” Problem

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The origin of the Baryonic Tully-Fisher Relationship (BTFR) $M_{\text{Bar}} \propto V^4$ remains an open quandary since empirically discovered thirty years ago. This relationship has been considered an exact function of galactic dynamics underpinning Modified Newtonian Dynamics (MOND) or a $\Lambda$CDM baryonic nuisance constraint. Since the BTFR is a central tool in astrophysical research, it is prudent to fully understand its origin and basis for existence.

This work leverages recent availability of extended, “complete” rotation curves for the MW and M31 exhibiting significant velocity declines just beyond their galactic disks. By modeling outer velocities as Keplerian, galactic dynamic mass is equal to $RV^2/G$, where $R$ is disk radius, and $V$, rotational velocity. This equation is also governed by the general Newtonian relationship $R \propto M_{\text{Dyn}}/V^2$ and a dynamical relationship, $R \propto V^2$. This constrains galaxy morphologies to a narrow band of physically permissible $R-V$ for any given dynamic mass $M_{\text{Dyn}}$ and directly leads to the fundamental relationship $M_{\text{Dyn}} \propto V^4$. This is linked to baryonic mass (BTFR) via the ratio of dynamic/baryonic mass considered constant for the entire disk galaxy population, recognized as a foundational element to explain the BTFR. Note that the BTFR only relates the baryonic mass component to galactic rotation, and not angular momentum and energy contributions. This model is superior to mainstream propositions that rely on unproven beliefs rather than a solid mechanical foundation. More importantly, this proposal is advantaged in that it is amenable to the scientific method and subject to falsification.

INTRODUCTION

A “state model” for rotationally supported galaxies is presented. The model is based on the application of simple mechanical law with respect to morphology and kinematics as obtained from very recent astrophysical data. Results indicate disk galaxies operate in a very confined window that serves as a “necessary precondition” to properly define the state (Curiel 2016). By definition, any dynamical galactic formation and evolution processes stemming from an incomplete or inaccurate state description must be suspect.

In this paper we report a significant breakthrough in establishing a basic physical model for total galactic dynamic mass ($M_{\text{Dyn}}$). The model is based on extended, accurate rotation velocity profiles for the Milky Way (MW) and the Andromeda galaxy (M31). We then extend this model to an irregular dwarf galaxy Andromeda IV (And IV) to span a wider range of galactic mass. The model is designated Rotation Curve-Spin Parameter (RC-SP) model and is based on a classical mechanical interpretation of dynamic mass. This approach does not introduce new hypotheses and defines galactic state parameters in terms of observables - rotation velocity, disk radius, and dynamic mass. To maintain convention between the RC-SP model and the two mainstream theories, $\Lambda$CDM and MOND, we define dynamic/baryon mass ratios as baryonic fraction (fb) and mass discrepancy (D), respectively (Milgrom 2016).
We define each RC-SP parameter, and how as an integrated set, they are responsible for the phenomenon known as “missing mass.” The empirical specificity and precision of the BTFR faithfully reflects this mechanical solution relating rotation velocity ($V$) and disk radius ($R$) if and only if dynamic mass is treated as a point mass equivalent (e.g., Keplerian decline beyond the galactic disk proper). In following sections, we take a step-by-step approach in how each RC-SP galactic parameter is determined.

**DYNAMIC MASS ($M_{\text{Dyn}}$)**

A main issue for $\Lambda$CDM (and MOND) is the inability to properly define a galaxy’s observed dynamic mass properties in pure physical terms. RC-SP overcomes this deficiency by obtaining total galactic dynamic mass directly from extended rotation curves without need for further interpretation. Figure 1 and 2 demonstrate the process for the MW using two independent literature sources (Sofue 2015).

Figure 1 defines three RC-SP critical parameters; average rotation velocity ($V$), break-point radius where Keplerian decline becomes apparent ($R$), and (baryonic and dynamic) mass specified by the Keplerian profile ($M_{\text{Dyn}}$). All needed RC-SP galactic parameters are obtained from this single plot using the following three-step process:

1. Establish galactic dynamic mass (green dash) by modeling the outer galactic rotation curve as a conventional Keplerian “point-mass” equivalent,
2. Estimate average disk “flat” velocity (blue-dash) and extend this velocity until it intersects with the Keplerian curve,
3. Obtain associated disk radius (black-dash).

The RC-SP radius ($R$) is closely related to the outer “edge” of the HI gas disk where the break-point in velocity occurs. This is fortuitous as HI gas tends to accurately track gravitational potential well beyond the stellar disk proper (Lelli 2016). Using data from another survey, Figure 2 returns the same RC-SP parameters obtained in the previous figure (Huang 2016). We can be confident that these parameters
truly reflect the physical state of the MW.

![Observed Milky Way Rotation Curve](image)

**Figure 2:** MW rotation curve from a second source indicating an average flat rotation velocity of 230 km/s (horizontal blue dash) transitioning to a Keplerian decline (green dash) at 40 kpc (black dash). The dynamic mass is estimated at $5 \times 10^{10} M_\odot$ (green dash), as in Figure 1. Image source - Fig. 11 (Huang 2016)

The above figures exhibit a wide spectrum of velocity-radius combinations that can satisfy the observed dynamic mass. Although the MW exhibits a particular combination (230 km/s and 40 kpc), there are infinite velocity-radius combinations possible raising two serious concerns: Why does the MW exhibit this particular combination of rotation velocity and radius and not others? Are all disk galaxies similarity constrained to specific velocity-radius combinations similar to what is observed for the MW? These two questions must be answered in order to fully appreciate the mechanical solution proposed herein.

Next, we apply the same process to M31 and demonstrate the observed Keplerian decline is a universal feature for massive bright spirals. We then extend this approach to irregular dwarf galaxy And IV to demonstrate the universal nature of these galactic parameters. For example, Figure 3 provides the extended rotation curve for M31 from the same source that provided Figure 1 (Sofue 2015). In the figure below, M31’s RC-SP parameters are determined from the physical rotation curve.
Sofue derived an average rotation velocity that is significantly lower than 265 km/s selected for the RC-SP parameter. Rather, we rely on the rotation curve shown in the inset from which M31’s dynamic mass was estimated at 10x10^{11} M☉ (Chemin 2009). Curiously, Sofue’s virial mass estimate (r<200 kpc) M_{200} = 13.9±2.6x10^{11} M☉ is close to the RC-SP value of 14x10^{11} M☉. This simple example illustrates the state of uncertainty and confusion existing today related to this fundamental galactic parameter. By defining dynamic mass in physically observed concrete terms, RC-SP allows little room for wide-ranging estimates plaguing theoretically-based “missing mass” models. This precision is demonstrated as we apply The RC-SP prescription to an irregular dwarf galaxy Andromeda IV (And IV). This galaxy had its rotation curve measured to just beyond peak velocity, and includes an estimate of baryonic and total dynamic mass (Karachentsev 2015). From these measures, a complete physical description of the galaxy is possible under RC-SP, as presented in Figure 4 below.

Figure 4: Andromeda IV RC-SP rotation velocity (blue dash), disk radius (vertical black dash) and Keplerian velocity profile (green dash) are shown in relationship to the rotation.
Figure 4 presents measured rotation velocity data on the left, extending to ~8 kpc. From the data, the rotation curve has peaked and may be transitioning to a decline beyond that radius. The same study also provided a total dynamic mass estimate equal to $0.034 \times 10^{11} \text{M}_\odot$ (D=9.1, fb=0.11). This situation provides a unique opportunity to demonstrate the utility of RC-SP over a wide range of galactic mass.

In the above figure, the observed dynamic mass is modeled as a point-mass equivalent (green-dash) according to the three-step process described above. As with the two massive spirals, the average (peak) velocity is proportional to $V_{\text{Peak}}/V_{\text{Bar}} = \sqrt{D}$. The intersection of $V_{\text{Peak}}$-$M_{\text{Dyn}}$ establishes the disk radius (R). The open triangle denotes the singular solution meeting all three physical constraints defining this dwarf galaxy. Interestingly, a recent comparison of cold dark matter N-body simulations to observation indicate less massive galaxies ($V<100 \text{ km/s}$) do not have the highly extended HI gas disk radii that is commonly assumed (Maccio 2016). Modeling the dynamic as a Keplerian decline suggests And IV's HI gas disk extends to 8 kpc and not far beyond.

In the next section, we apply the concept of spin parameter to equate disk galaxy morphology and kinematics to angular momentum and total energy (from which baryonic content can be derived). We quantify these properties for our three example galaxies utilizing RC-SP derived galactic parameters and extend these findings to the general disk galaxy population. This approach is motivated by a recent investigation into galactic surface density distribution (Marr 2015).

**ANGULAR MOMENTUM (J) AND TOTAL ENERGY (E)**

The RC-SP model is based on classical methods and heavily relies on the dimensionless spin parameter equation. This equation relates the contribution of angular momentum and energy to the total dynamic mass of the galaxy. The Peebles derivation is given below (Peebles 1969):

$$\lambda = \frac{J\sqrt{E}}{GM_{\text{Dyn}}^{7/2}}$$

Where $J = \text{Angular Momentum}$, $E = \text{Total Energy}$, $M_{\text{Dyn}} = \text{Dynamic mass}$, and Total $E = K + PE$.

The above equation is typically not solved as a stand-alone equation due to difficulty in quantifying $J$ and $E$. To overcome this deficiency, an alternative form of the spin parameter was developed by Bullock to dimension the spin parameter equations with direct observables; rotation velocity and disk radius (Bullock 2001). This alternative is consistent with Peebles under the added constraint of an isothermal halo and extended flat rotation velocity:

$$\lambda' = \frac{J}{\sqrt{2M_{\text{Dyn}}VR}}$$

Where $J = \text{Angular Momentum}$, $M_{\text{Dyn}} = \text{Dynamic Mass}$, $V = \text{Rotation Velocity at radius } R$.

It has been shown that Peebles and Bullock equations are meaningfully equivalent and can be equated (Knebe 2008):

$$\frac{J\sqrt{E}}{GM_{\text{Dyn}}^{7/2}} = \frac{J}{\sqrt{2M_{\text{Dyn}}VR}}$$

Solving for $E$:

$$E = \frac{G^2M_{\text{Dyn}}^3}{2V^2R^2}$$
J and E are determined by inserting RC-SP defined rotation velocity (V), disk radius (R) and dynamic mass (MDyn) with results shown in Table 1 for the Milky Way, M31, and And IV. The last column provides the total galactic dynamic mass using the generalized model discussed in later in this section.

<table>
<thead>
<tr>
<th>RC-SP Galactic Parameters</th>
<th>M_{Dynam}</th>
<th>V (km/s)</th>
<th>R (kpc)</th>
<th>\log M_{\text{E}}</th>
<th>\log M_{\text{J}}</th>
<th>\log M_{\text{Dyn}}=R^2/G</th>
</tr>
</thead>
<tbody>
<tr>
<td>M31 Spiral</td>
<td>14</td>
<td>265</td>
<td>90</td>
<td>17.0</td>
<td>16.1</td>
<td>14.7</td>
</tr>
<tr>
<td>Milky Way Spiral</td>
<td>5</td>
<td>230</td>
<td>40</td>
<td>16.1</td>
<td>15.5</td>
<td>4.9</td>
</tr>
<tr>
<td>And IV – (obs.)</td>
<td>0.034</td>
<td>45</td>
<td>7.5</td>
<td>12.8</td>
<td>11.6</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Table 1: RC-SP galactic properties by column; (1) dynamic mass directly from extended rotation curve, (2) “average” disk velocity and (3), disk radius at the Keplerian decline “break-point” in rotation curve. Columns (4, 5) represent RC-SP total energy and angular momentum for a constant spin parameter \( \lambda = 0.423 \). Column (6) is the general RC-SP derived “state” equation linking galactic physical morphology and kinematics to dynamic mass.

To determine the veracity of the RC-SP definitions, in Figure 5, we plot E and J against dynamic mass and to compare Marr’s results encompassing a wide variety of galaxies and find them indistinguishable. Both independently obtained data sets argue for galactic log normal surface densities rather than the simple “single component exponential distributions” often used for the distribution of the entire distribution of the galactic disk (CGM notwithstanding). The importance of a log normal surface density is that it physically permits galactic self-gravitation and naturally flat rotation profiles.

![Image](image-url)

Figure 5: Log E-J/MDyn relationship based on galactic log normal surface density distribution. Solid circles for MW and M31 are RC-SP E-J values and observed dynamic mass as obtained from rotation curves. This data supports the observed mean spin parameter \( \lambda = 0.423 \). Image source – top panel, Fig. 3 (Marr...
Figure 5 verifies RC-SP dynamics conforms to Marr, lending strong support for log normally distributed galactic surface densities. The precision fit between the two data sets is due to highly constrained spin parameter ($\lambda = 0.423 \pm 0.014$). In this analysis, Marr (and RC-SP) defines surface density in terms of the total contribution, baryons, angular momentum and energy (not just stellar and/or gas component often used to model exponential distributions). It is a simple exercise to sum individual surface density contributors to arrive at a total surface density that can be accurately described by a particular log normal distribution. Similar to Marr’s dynamic mass equation, RC-SP also links galactic dynamic mass to disk radius and rotation velocity with $1/G$ as the constant of proportionality:

$$M_{\text{Dyn}} = \frac{RV^2}{G}$$

Where $R$ is the disk radius, $V$ is the RC-SP rotational velocity and Newton’s constant, $G$.

Column (6) in Table 1 provides the modeled dynamic mass estimates for the MW and M31 based on the above equation. Within 60 kpc, the RC-SP MW dynamic mass, $M_{\text{Dyn}} = 4.9 \times 10^{11} M_\odot$ is comparable to an earlier well-established dynamic mass estimate equal to $4.0 \pm 0.7 \times 10^{11} M_\odot$ (Xue 2008). Both are consistent with another recent estimate, $4.25 \times 10^{11} M_\odot$ obtained by treating galactic “missing mass” as a perfect fluid composed of dark matter (Potapov 2016). There is no doubt that RC-SP parameter definitions accurately describe the MW. Rules of simple proportionality require nearly all other rotationally supported galaxies also follow the RC-SP prescription.

With a slight transformation, it is not coincidental that the RC-SP dynamic mass and Newton’s general equation for circular velocity are equivalent:

$$v = \sqrt{GM/r}$$

Ironically, dark matter was initially introduced to reconcile galactic rotation curves with Newtonian mechanics only to have it removed! $\Lambda$CDM’s issue is one of timing. Dark matter solutions became popular in the 1970’s based on incomplete and inaccurate galactic rotation curves. As such, $\Lambda$CDM proponents built much of their theory from incomplete data sets, including flat velocities to the outer edge of the dark matter halo (~ 200-400 kpc, depending on $\Lambda$CDM model).

The next section focuses on galactic baryonic content using mainstream definitions - $\Lambda$CDM baryonic fraction ($f_b$) and MOND mass discrepancy ($D$). To this point, we have incorporated the spin parameter to define dynamic mass in terms of angular momentum and total energy. If it was not already apparent, it is evident that galactic baryons are only part of the RC-SP solution. In this galactic model, baryons can be considered “along for the ride,” as their morphology and kinematics are completely determined from the formation process (baryons reveal the absolute magnitude of galactic total mass). Angular momentum and energy is specific to baryonic mass, and as baryonic content increases, so does total dynamic mass in direct proportion.

**BARYON MASS (MBAR)**

This section establishes “fiduciary” baryonic mass estimates for our three example galaxies. For convenience, we fix the dynamic/baryon mass ratio to the $\Lambda$CDM cosmic average $f_b = 0.17$. By fixing the baryonic fraction to the cosmic average, dynamic mass is not added or removed from the universe maintaining the balance determined from the cosmic microwave background and other sources (a precise baryonic content is not critical to obtaining the holistic RC-SP solution).

Estimating the baryon content (stars, gas, etc.) of disk galaxies is a continually improving field. The general trend is to add additional baryonic mass to the galactic inventory, effectively removing it from the “missing mass” deficit. One recent example is the discovery of the Circum-Galactic Medium (CGM),
a massive halo embedding the disk into an estimated stellar disk’s worth of baryonic mass. This discovery effectively “closes the Galaxy’s baryon census” justifying the fiduciary cosmic average value (Nicastro 2016). Table 2 shows the current baryon census by component for each of our three example galaxies:

![Table 2: Milky Way and M31 fiducial baryonic inventory fixed at fb=0.17 in relation to the RC-SP determined dynamic mass. And IV - observed values (Karachentsev 2015)]

In the lower portion of Table 2, dynamic/baryon mass ratios are reported for both mainstream paradigms through the identity \( D \equiv 1/f_b \). This table is only included for completeness as RC-SP model is driven from total dynamic mass, not singled out components as is often the case.

The next section offers strong support for a mean baryonic fraction governing disk galaxies, independent of baryon content. This constraint directly contradicts ΛCDM claims of a non-linear relationship between dark matter halo mass (~dynamic mass) and baryon content. This basic ΛCDM tenet was formulated before accurate assessments of galactic baryonic and dynamic mass were available and has been confirmed to be in error.

**BARYONIC FRACTION (MDYN/MBAR)**

Unlike ΛCDM or MOND, RC-SP relies exclusively on empirical data. In this regard, RC-SP has more in common with MOND as it is baryonically based and requires a mean baryonic fraction, independent of galactic mass. Figure 6 provides incontrovertible proof for constant galactic baryonic fraction independent of rotation velocity (dynamic mass proxy).
Figure 6: Observed baryonic fraction for a wide range of observed galactic rotation velocities ($\propto$ dynamic mass). Open circles represent RC-SP MW and M31 RC-SP galactic parameters. The open triangle is the observed value for irregular dwarf galaxy And IV ($f_b=0.11$ and $V\approx45$ km/s). For practical purposes, $R_1$ is equivalent to RC-SP disk radius $R$. Image source - Fig.7 (Bradford 2015)

Data presented in Figure 6 indicate a mean baryonic fraction clustering near the cosmic average for an extremely wide range of rotation velocities at odds with $\Lambda$CDM theory. The relatively large scatter is due to the use of baryons to define the fraction, exclusive of angular momentum and energy.

It has been known for almost twenty-years that the empirically tight BFTR relationship is only physically possible if disk galaxies are governed by a cosmically “universal” baryon fraction (McGaugh, The Baryon Fraction Distribution and the Tully-Fisher Relation 1997). With this knowledge, we can extend the RC-SP approach under the two following constraints; adherence to Newtonian mechanics, and total galactic dynamic mass treated as a Keplerian “point-mass” equivalent (e.g., the classic definition of a “particle”). We see these constraints directly apply within a few kpc beyond the galactic disk (~ HI gas radius). Inside this radius, disk dynamics dominate and are responsible for the observed dynamic mass as determined from extended rotation curves.

In the next section, we find these constraints lead to an analytical solution to the origin of the BTFR, $M_{\text{Bar}} \propto V^4$.

**THE BARYONIC TULLY-FISHER RELATIONSHIP**

The BTFR is a robust empirical correlation between estimated baryonic mass and observed (flat) rotational velocity depicted in Figure 7 (McGaugh 2011).
Figure 7: RC-SP dynamic mass – rotational velocity relationship (green dash). Data denotes observed baryonic mass for gas-rich galaxies. MW, M31 and And IV dynamic (open symbol) and baryonic (closed symbol) masses included for comparison to data set. McGaugh fits (black solid and red dash) with extended RC-SP fit (black dash). Image source - Fig. 3 (McGaugh 2011)

Figure 7 data represents gas-rich galaxy properties in the lower end of the galactic mass spectrum. Note that RC-SP derived data is entirely compatible with McGaugh’s BTFR fit. This figure serves as a template to illustrate the functional RC-SP relationship (open circles) derived from the dynamic mass and rotation velocity estimates. The magnitude of the offset (D=5.9) from McGaugh’s fit corresponds to a baryonic/dynamic mass ratio equal to the cosmic average fb=0.17. Rather than attempting to measure baryon content directly, RC-SP provides baryonic estimates that again are indistinguishable from the raw data with the MW and M31 data extending McGaugh’s fit to the range populated massive star-dominated spirals. This is evidence that RC-SP accurately describes the physical “galactic state” without resorting to “external” rotational support, normally ascribed to dark matter or modified Newtonian dynamics. These physical interrelationships are difficult to grasp from equations and best shown graphically in Figure 8 below.
Figure 8: RC-SP combined (R, V & M_{Dyn}) physical galaxy map. Individual gas-rich galaxy data from Table 1 (McGaugh, The Baryonic Tully-Fisher Relation of Gas Rich Galaxies as a Test of LCDM and MOND 2011) converted to M_{Dyn} via constant baryonic fraction f_b=0.17 [see Appendix]. This data confirms all disk galaxies are fully self-gravitating and exhibit dynamic masses that can be modeled as Newtonian point mass equivalents in the far field. The figure demonstrates galaxies follow a remarkably strict R-V relationship determining total galactic dynamic mass.

In the above figure, most of McGaugh’s gas-rich galaxies are plotted. McGaugh’s baryonic data was converted to galactic dynamic mass by multiplying total baryonic content by D=5.9 (conversely, M_{bar} can be divided by f_b=0.17 to arrive at the same dynamic mass estimate). Galactic radii were determined from the RC-SP model R=GM_{Dyn}/V^2. Per Figure 8, for a given disk radius (R), galactic dynamic mass (M_{Dyn}) varies inversely to the square of rotation (V). A complementary relationship ties disk radius to the square of the ration velocity. These galactic constraints conspire to create the “effective” relationship M_{Dyn} \propto V^4:

\begin{align*}
R \propto V^2 \quad \text{and} \quad R \propto \frac{M_{Dyn}}{V^2} = M_{Dyn} \propto V^4
\end{align*}

The BTFR is then trivially obtained,

\begin{align*}
\text{BTFR: } \frac{M_{Dyn}}{D} \propto M_{gas} \propto V^4
\end{align*}

Figure 8 reveals a simple relationship between disk radii, rotation velocity and dynamic mass meeting all three imposed physical constraints. Individual galaxy data is summarized in the Appendix including a dynamic mass error analysis between analytic RC-SP model and physical placement in Figure 8 directly from R and V values. We find good agreement with less than twenty-percent offset between the two suggesting a systematic effect due to parameter definitions or perhaps a galactic feature such as pseudo-point mass velocity profiles as opposed to the Newtonian model.

As this exercise demonstrates, the RC-SP model is internally and externally self-consistent and obeys all imposed classical mechanics constraints, a statement neither \Lambda CDM nor MOND proponents can
convincingly make.

**COSMOLOGICAL IMPLICATIONS**

An issue remains in the cosmology required to impart disk galaxies with universally constrained morphological and kinematic properties. Two possibilities exist; loosely constrained galaxy formation with natural selection over the course of cosmic history permitting only those galaxies having specific combinations to survive, or an early era, cosmic event resulting in shared galactic properties. The first possibility has been discredited with studies demonstrating both baryonic fraction and spin parameter have remained stable throughout most of observed history (Cervantes-Sodi 2012). The second possibility is strongly favored and argues for a “common ancestry” as opposed to “survival of the fittest.”

**CONCLUSION**

A galactic state model is presented based on a novel technique that expresses total dynamic mass in precise physical kinematic terms, angular momentum and energy or morphologically via radius and rotation velocity. This model employes Newtonian mechanics to solve the problem of galactic “missing mass,” and in doing so, reveals the physical origin of the BTFR. We demonstrate the empirical BTFR is a manifestation of baryonic physics in the disk galaxy setting. This physically and mechanically grounded based solution presents an alternative to ΛCDM and MOND models.

**APPENDIX**

Galaxy Data (McGaugh 2011) and Associated RC-SP Parameters

**BIBLIOGRAPHY**


